Output of a Sound Source in a Reverberation Chamber and Other Reflecting Environments*

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It is important to know the output of a source as a function of position in a reverberation chamber. Expressions are given for the sound power output of simple monopole and dipole sources as functions of source position in various reflecting environments. They are obtained by the use of the method of images and a general formula due to Rayleigh for the output of a number of simple sources. The cases of a dipole source near a reflecting plane and a simple source near a reflecting edge and corner are treated, and the effect of the band width of the source is considered. The results apply when the reflectors enclose the source, as in a reverberation chamber, unless the distance in wavelengths between parallel walls is small and the absorption in the enclosure is low. Some experimental data are given, and the reverberation chamber method of measuring the power output of sources is discussed. In the general case of an extended source emitting nonspherical waves near reflecting surfaces, it may be more convenient to find the variation of power output with position by experiment than by calculation.

INTRODUCTION

An important property of sound sources is that their sound power output and directional patterns in general vary when they are placed in different positions near reflecting surfaces.

The effect is caused by the sound reflected from the reflectors reacting back on the source. This reaction can be considered as an impedance reflected onto the source. The reflected impedance varies with the environment, and thus in general the sound power output of the source varies with the environment.

The effect is discussed in the literature,[1–3] and Fischer[4] has given expressions for the dependence on source-reflector separation for a simple source near a plane. In this paper expressions are given for various source-reflector systems that are of interest in architectural acoustics, including the case of a dipole source near a reflecting plane, and a simple source near a reflecting edge and corner.

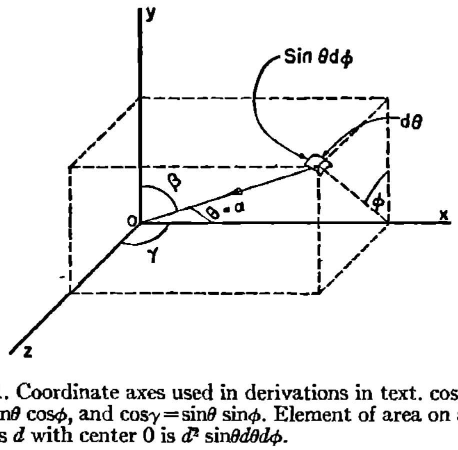

FIG. 1. Coordinate axes used in derivations in text. $\cos\alpha=\cos\theta$, $\cos\beta=\sin\theta\cos\phi$, and $\cos\gamma=\sin\theta\sin\phi$. Element of area on a sphere of radius d with center 0 is $d^2\sin\theta d\theta d\phi$.

These expressions follow from the use of the method of images and a formula due to Rayleigh for the output of a number of simple sources. It is shown that, in general, the power output of the source differs significantly from the free-field value if the distance of the source from the reflector is less than the wavelength of the sound emitted.

These results are of practical importance in several ways. For example, they show where a source must be located in a reverberation chamber if the free-field value of the power output of the source is to be measured. They also enable one to estimate to what extent noise can be reduced or increased by manipulating reflectors near a source. Such manipulation of reflectors could also be used to match a source into a medium.

We first obtain expressions for the power output variations of some simple types of source and reflector, and then consider their application to the more complicated source-reflector systems met in practice.

OUTPUT OF SOURCE NEAR PLANE REFLECTOR

Rayleigh's formula[5–8] for the directional pattern DP of a number of simple sound sources of uniform strength and frequency, placed at arbitrary positions in space, can be written

$$DP\propto\sum_m\sum_n\cos(kd_{mn}+\epsilon_{mn}),\qquad(1)$$

where $k=\omega/c_0$, $\omega=2\pi f$, c_0 is the sound velocity, and f is the frequency of the sound sources.

* A lecture on this topic was given at a meeting of the Acoustical Society of America on December 15, 1955.

1 Lord Rayleigh, *Theory of Sound* (Dover Publications, New York, 1945), Vol. 2, pp. 108–112.
2 H. Lamb, *Dynamical Theory of Sound* (Edward Arnold and Company, London, 1931), second edition, pp. 229 and 233.
3 L. L. Beranek, *Acoustics* (McGraw-Hill Book Company, Inc., 1954), pp. 319–320.
4 F. A. Fischer, Elek. Nachr.-Tech. 10, 19 (1933).
5 Rayligh, *Collected Papers* (Cambridge University Press, London, 1912), Vol. 5, p. 137.
6 H. Stenzel, Elek. Nachr.-Tech. 6, 165 (1929).
7 H. Stenzel, *Leitfaden zur Berechnung von Schallvorgaengen* (Berlin, 1939) (English translation by A. R. Stickley, published by Naval Research Laboratory, Washington, D. C.).
8 E. Rhian, J. Acoust. Soc. Am. 26, 704 (1954).

The path difference of rays from sources m and n in the direction with direction cosines $\cos\alpha$, $\cos\beta$, $\cos\gamma$ (see Fig. 1) is d_{mn}, i.e.,

$$d_{mn} = (x_m - x_n)\cos\alpha + (y_m - y_n)\cos\beta + (z_m - z_n)\cos\gamma, \quad (2)$$

where (x_m, y_m, z_m) are the position coordinates of the mth source. ϵ_{mn} is the phase difference between sources m and n. The term "directional pattern" is defined here as the part varying with direction in the expression for the mean squared pressure in the far field of a source.

The sound output W of all the N sources is obtainable by integrating (1) over a sphere, and can be written

$$W/W_f = N + 2\sum(\cos\epsilon_{mn})\, j_0(r_{mn}'), \quad (3)$$

where the sum is taken over every pair of sources. If only one source is real, the rhs of (3) is divided by N. $W_f = 2\pi k^2 \rho_0 c_0$ is the free-field power output of one source. The spherical Bessel function is $j_0(r_{mn}') = (\sin r_{mn}')/r_{mn}'$, and $r_{mn}' = 4\pi r_{mn}/\lambda$, i.e., 4π times the distance in wavelengths between sources m and n.

$$r_{mn} = [(x_m - x_n)^2 + (y_m - y_n)^2 + (z_m - z_n)^2]^{\frac{1}{2}}. \quad (4)$$

From these formulas we can obtain comparable expressions for the output of a single source near certain reflecting surfaces, by the use of the method of images.[2,9] In this method, sound reflected by the reflector is considered to emanate from one or more image sources. These image sources are placed so that their radiation combines with that of the object source to satisfy the boundary conditions of the problem.

We consider that the reflectors are rigid and 100% reflecting, so the boundary condition is that on the reflecting plane the normal component of particle velocity is zero. The strength, i.e., peak volume velocity, of the source is fixed, and independent of the environment of the source.

For the case of a simple source near a plane reflecting surface, as shown in Fig. 2(a), one image source is enough to satisfy this boundary condition, the image source being equal in strength and phase to the object source.

The directional pattern of the source plus reflector is then, from (1)

$$DP \propto \cos^2 a, \quad (5)$$

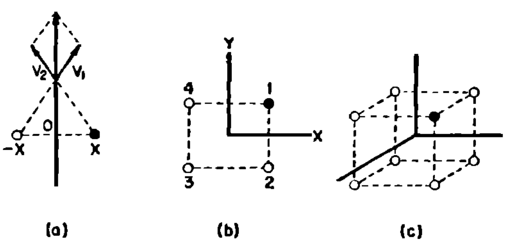

FIG. 2. ● Point source. ○ Image source. Acoustic images at one-wall, two-wall, and three-wall reflectors when one point source is present. The black lines represent the profiles of the reflectors.

[9] H. Lamb, *Hydrodynamics* (Dover Publications, New York, 1945), p. 129.

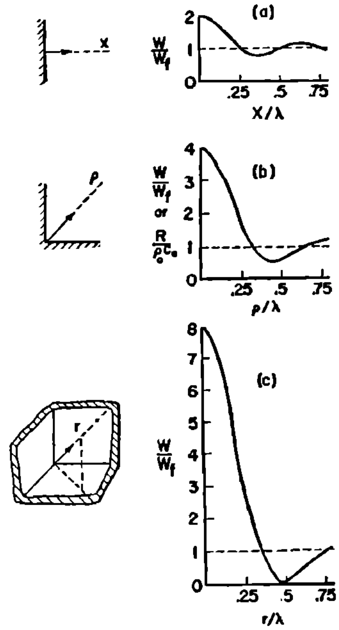

FIG. 3. Theoretical curves for the relative power output W/W_f of a simple source as a function of position near one-, two- and three-plane reflectors at right angles. ρ and r are the distances of the source from the origin along the lines of symmetry, and $k = \omega/c_0$ is the wave number. When the source is many wavelengths away from the reflectors, its power output W approaches the free field value W_f asymptotically. The dashed lines are the asymptotes. See Table II for the expressions plotted here.

where $a = kx\cos\alpha$; the reflecting plane is the yz plane of Fig. 1, and the source is a distance x from it.

The power output W of the source plus reflector[4] is found from (3) to be

$$W/W_f = 1 + j_0(x'). \quad (6)$$

W_f is the output of the source in a free field, for example, suspended on a string in an anechoic chamber. The spherical Bessel function $j_0(x') = (\sin x')/(x')$, and $x' = 4\pi x/\lambda$, i.e., 4π times the distance in wavelengths from source to reflector.

The power output of the source-reflector combination can also be obtained by calculating the impedance seen by the source, as shown in Appendix II.

Expression (6) is plotted in Fig. 3(a). It can be written in terms of the other variables f, the frequency, or k, the wave number, as

$$W/W_f = 1 + j_0(2kx) \quad (7)$$

$$= 1 + j_0(4\pi f x/c_0). \quad (8)$$

The form (6) is used here since it is compact and helps to underline the fact that the position in space of the source, measured in wavelengths, is the important variable here.

SOURCE OF ARBITRARY DIRECTIONAL PATTERN

For a sound source of arbitrary directional pattern $A_1 = A(\theta,\phi)$, the method of images can still be used, but the image sources must now have particular directional patterns. If the source is distant x from an infinite rigid reflecting plane, an image source with directional pattern $A_2 = A(\pi - \theta, \phi)$ is needed to satisfy the boundary condition.

From (1) we then obtain for the directional pattern DP of source and reflector

$$DP \propto A_1^2+A_2^2+2A_1A_2 \cos 2a. \qquad (9)$$

The power output of the combination is then proportional to

$$\int_0^{\pi/2}\int_0^{\pi} (A_1^2+A_2^2+2A_1A_2 \cos 2a) \sin\theta d\theta d\phi, \qquad (10)$$

using the coordinate system of Fig. 1. The constant of proportionality is determined by normalization. This expression can be calculated if the functions $A_1=A(\theta,\phi)$ and $A_2=A(\pi-\theta,\phi)$ are given.

As an example, we take $A_1=\cos\theta$, using the coordinate system of Fig. 1, yz being the reflecting plane. This gives the source a figure-of-eight directional pattern like that of a dipole source in a free field with the dipole axis parallel to the x axis. When the source is distant x from the plane reflector, the directional pattern of the combination is found from (9) to be

$$DP \propto \cos^2\alpha \sin^2 a, \qquad (11)$$

and the power output from (10) is given by

$$W/W_f = 1-j_0(x')+2j_2(x'), \qquad (12)$$

where W_f is the free-field value of the power output of the source, $x'=4\pi x/\lambda$, $j_0(x')=(\sin x')/x'$, $j_1(x')=[j_0(x')-\cos x']/x'$, and $j_2(x')=(3/x')j_1(x')-j_0(x')$.

Equation (12), which is the same as Eq. (21) given later in this paper, is plotted in Fig. 4.

EFFECT OF BAND WIDTH

The foregoing results apply to sources emitting sound at one frequency only. For sources emitting sound at a few discrete frequencies, i.e., a line spectrum, the resultant effects are obtained by summing arithmetically the effects at the various single frequencies. For sources emitting continuous bands of noise, analogous results can be found by an integration, as follows.

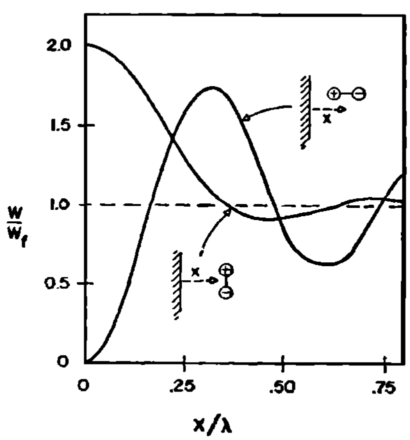

FIG. 4. Theoretical curves for the relative power output W/W_f of a dipole source *versus* distance x/λ in wavelengths from a plane reflector. Two cases are shown, with the dipole axis normal and parallel to the reflector. See Eqs. (21) and (19). The dashed line is the asymptote.

For a point source emitting noise with a continuous band width f_1 to f_2, (5) becomes

$$DP \propto [1/(f_2-f_1)]\int_{f_1}^{f_2} \cos^2 a\, df, \qquad (13)$$

$$\propto 1+[(\sin Bf_2-\sin Bf_1)/B(f_2-f_1)], \qquad (14)$$

where $B=(4\pi x \cos\theta)/c_0$.

The relative power output of the source, from (6) and (8) is

$$W/W_f = 1+[1/(f_2-f_1)]\int_{f_2}^{f_1} j_0(4\pi fx/c_0)df. \qquad (15)$$

The integral is the sine integral, which is tabulated. Graphs of (15) for band widths of ±10%, one octave, and 5 octaves are given in Fig. 5. It is apparent that even with the wide band of 5 octaves the main feature of the curve remains.

Dipole Source

Expressions for the directional pattern and power output of a dipole source as functions of distance from a plane rigid reflector are obtainable from the general formulas (1) and (3).

When the dipole is in the xy plane of Fig. 1, and its axis makes an angle η with the reflector (yz plane), its directional pattern is

$$DP \propto \sin^2\eta \cos^2\alpha \sin^2 a + \cos^2\eta \cos^2\beta \cos^2 a, \qquad (16)$$

and its power output W in terms of W_f, the free-field output of the dipole, is

$$(W/W_f)=1+(3/x')j_1(x') +3\sin^2\eta[(1/x')j_1(x')-j_0(x')], \qquad (17)$$

where $j_1(x')=[j_0(x')-\cos x']/x'$.

When the dipole axis is parallel to the plane, $\eta=0$ and

$$DP \propto \cos^2\beta \cos^2 a \qquad (18)$$

$$(W/W_f)=1+j_0(x')+j_2(x'), \qquad (19)$$

where $j_2(x')=(3/x')j_1(x')-j_0(x')$.

When the dipole axis is normal to the plane reflector, $\eta=90°$ and

$$DP \propto \cos^2\alpha \sin^2 a \qquad (20)$$

$$(W/W_f)=1-j_0(x')+2j_2(x'). \qquad (21)$$

The foregoing expressions† are tabulated in Table I, and graphs of (19) and (21) are given in Fig. 4.

These results are of some practical interest. For

† Results (18) to (21) were given in a lecture on this topic to the Acoustical Society of America on December 15, 1955. More recently, they were given by U. Ingard and G. L. Lamb, Jr., J. Acoust. Soc. Am. 29, 743 (1957).

TABLE I. Expressions for the directional pattern and relative power output of a dipole source as functions of distance x from a plane reflecting wall.[a]

	Dipole makes angle η with wall	Dipole axis parallel to wall	Dipole axis normal to wall
Directional pattern, DP	$\sin^2\eta\cos^2\alpha\sin^2 a+\cos^2\eta\cos^2\beta\cos^2 a$	$\cos^2\beta\cos^2 a$	$\cos^2\alpha\sin^2 a$
Relative power output W/W_f	$1+(3/x')j_1(x')+3\sin^2\eta[(1/x')j_1(x')-j_0(x')]$	$1+j_0(x')+j_2(x')$	$1-j_0(x')+2j_2(x')$

[a] $a=kx\cos\alpha$; $x'=4\pi x/\lambda$; $j_0(x')=(\sin x')/x'$; $j_1(x')=[j_0(x')-\cos x']/x'$; $j_2(x')=(3/x')j_1(x')-j_0(x')$. In Fig. 1, the yz plane is reflecting, and the dipole is in the xy plane. See Fig. 4 for graphs of some of the expressions here for the relative power output.

example, at low frequencies a loudspeaker with no cabinet radiates like a dipole, as experimental results given later show.

OUTPUT OF SOURCE NEAR REFLECTING EDGE AND CORNER

When a point source is near 2 reflectors at right angles, 3 image sources are needed to satisfy the boundary conditions, as shown in Fig. 2(b).

The normal velocity components of the sound from sources 1 and 2, 3 and 4, cancel on the xz plane, and similarly for sources 1 and 4, 2 and 3, on the yz plane. Analogous arguments hold for the case of 3 reflectors at right angles, Fig. 2(c), where 7 image sources are needed.

Thus when the source is at (x,y,z) the images in the three reflector case lie at the points $(-x, y, z)$, $(x, -y, z)$, $(x, y, -z)$, $(-x, -y, z)$, $(x, -y, -z)$, $(-x, y, -z)$, $(-x, -y, -z)$, and analogously for the two-plane case.

Inserting these image positions in (1) and (3) we obtain the directional pattern and power output of a simple source near two-, and three-plane reflectors.

The directional pattern for a simple source at position xy near a reflecting edge [xz and yz planes reflecting, see Figs. 1 and 2(b)] is

$$DP\propto(\cos a\cos b)^2, \tag{22}$$

where $a=kx\cos\alpha$, $b=ky\cos\beta$, and the coordinate system and symbols of Fig. 1 are used.

The relative power output is

$$W/W_f=1+j_0(x')+j_0(y')+j_0(\rho'), \tag{23}$$

where $x'=4\pi x/\lambda$, $y'=4\pi y/\lambda$, and $\rho'=4\pi(x^2+y^2)^{1/2}/\lambda$.

If the source is confined to the line of symmetry from the origin, bisecting the angle xoy, its relative power output is

$$W/W_f=1+2j_0(\rho'/\sqrt{2})+j_0(\rho') \tag{24}$$

[see Figs. 3(b) and 9].

When the source is at position (x,y,z) near a reflecting corner the directional pattern is

$$DP\propto(\cos a\cos b\cos c)^2, \tag{25}$$

where a and b are as in (22) and $c=kz\cos\gamma$, (see Fig. 1).

The relative power output of the source is

$$W/W_f=1+j_0(x')+j_0(y')+j_0(z')+j_0(\rho_1')+j_0(\rho_2')+j_0(\rho_3')+j_0(r'), \tag{26}$$

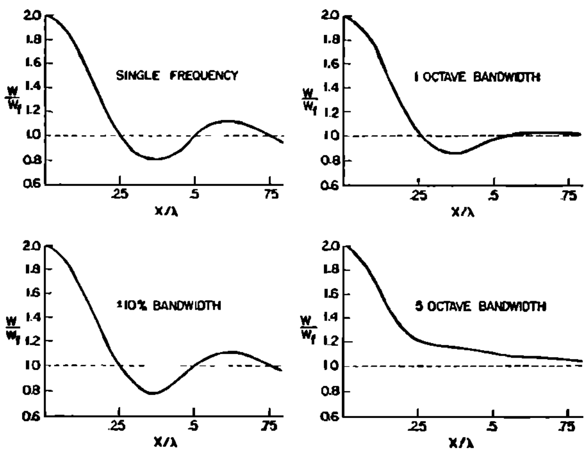

FIG. 5. Theoretical curves for the relative power output W/W_f of a simple source as a function of distance x/λ in wavelengths from a plane reflector, when the source vibrates with various band widths. In the abcissae $\lambda=(\lambda_1+\lambda_2)/2$ where λ_1 and λ_2 are the wavelengths at the extremes of the bands. The dashed lines are asymptotes.

where $\rho_1'=2k(x^2+y^2)^{1/2}$, etc., and $r'=2k(x^2+y^2+z^2)^{1/2}$.

When the source is confined to the line of symmetry from the corner, its relative power output is

$$W/W_f=1+3j_0(r'/\sqrt{3})+3j_0(r'\sqrt{\tfrac{2}{3}})+j_0(r'), \tag{27}$$

[see Fig. 3(c)].

The above results are tabulated in Table II, and some graphs are given in Figs. 3 and 6. The results were obtained by a different technique in a previous paper.[10] Contour maps of Eqs. (17) and (20) are being prepared with the aid of an electronic computer, and it is hoped to publish them in a later paper.

The curves in Fig. 3 are caused by interference, and depend only on the distance in wavelengths between source and reflector.

The power output variation can be considered to result from the source seeing a different radiation resistance R at different positions near the reflector, while the strength or volume velocity of the source remains the same. The ordinates in Fig. 3 can all be labeled R/ρ_0c_0, as in Fig. 3(b).

The curves can also be considered from the standpoint of the method of images. For example in Fig. 2(a), when the source touches the reflector, it concides with its image; its strength is thus doubled, and its power

[10] R. V. Waterhouse, J. Acoust. Soc. Am. 27, 247, 256 (1950).

TABLE II. Expressions for the directional pattern and relative power output of a simple source as functions of position (x,y,z) near 1-plane, 2-plane, and 3-plane reflectors at right angles.[a]

	1-plane reflector (yz plane)	2-plane reflector (xz, yz planes)	3-plane reflector (xy, xz, yz planes)
Directional pattern, DP	$\cos^2 a$	$(\cos a \cos b)^2$	$(\cos a \cos b \cos c)^2$
Relative power output, W/W_f	$1+j_0(x')$	$1+j_0(x')+j_0(y')+j_0(\rho')$	$1+j_0(x')+j_0(y')+j_0(z')+j_0(\rho_1')+j_0(\rho_2')+j_0(\rho_3')+j_0(r')$
	Along line of symmetry	$1+2j_0(\rho'/\sqrt{2})+j_0(\rho')$	$1+3j_0(r'/\sqrt{3})+3j_0(r'\sqrt{\tfrac{2}{3}})+j_0(r')$

[a] $a=kx\cos\alpha$, $b=ky\cos\beta$, $c=kz\cos\gamma$. $\cos\alpha=\cos\theta$, $\cos\beta=\sin\theta\cos\phi$, and $\cos\gamma=\sin\theta\sin\phi$. $\rho_1'=2k(x^2+y^2)^{\frac{1}{2}}$ etc. $r'=2k(x^2+y^2+z^2)^{\frac{1}{2}}$. $j_0(x')=(\sin x')/x'$. See Fig. 1 and Fig. 3 for graphs of the relative power-output functions above.

output integrated over a whole sphere would be quadrupled. However, only the radiation over $\frac{1}{2}$ a sphere can exist, i.e., in front of the plane wall, and the power output is thus double, not quadruple, the free-field value.‡

For similar reasons, the power output of a simple source goes up to 4 times the free-field value when the source is near a reflecting edge and up to 8 times near a reflecting corner.[11]

Figure 7 shows the interference of the sound-pressure waves from two point sources. The lower part of the figure shows the mean pressure distribution for each source along the line joining the two sources.

When the distance between the sources is less than one wavelength, the high intensity parts of the fields interact, and the total energy present is strongly dependent on the source separation. When the distance between the sources is a few wavelengths, the high-intensity central part of the field of each source is little affected by the interference of the weak outer field of the other source. As the separation of the sources increases, the effect of interference becomes negligible, and the power output of each source approaches the free-field value, as shown in Fig. 3(a).

One feature of the curves in Fig. 3 is that for certain source positions the power output is less than the free-field value. This occurs in all the regions of the graphs in Fig. 3 where the curve is below the dashed lines. In such regions the interference is predominantly destructive, and the power output of the simple source is not increased but diminished.

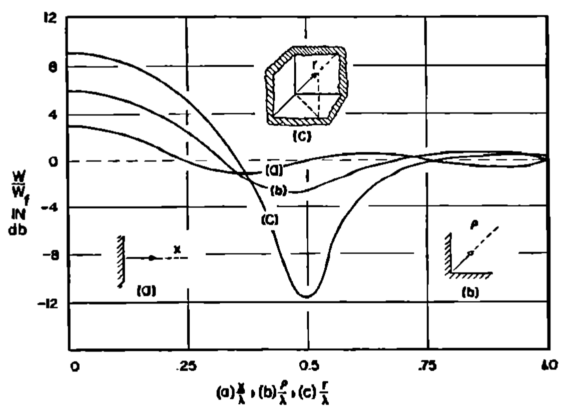

FIG. 6. Decibel plot of curves in Fig. 3.

For example, in Fig. 2(c) the output is less than the free-field output when the source distance r from the corner is in the region $0.36\lambda < r < 0.78\lambda$. If the source is fixed at a distance $r=1$ ft from the corner, the corresponding frequency range over which the output is diminished is approximately the octave 400–800 cps.

Figure 3(c) shows the interesting fact that the destructive interference of spherical waves near a reflecting corner can be almost complete. This is perhaps surprising at first sight, in view of the different geometry of the systems formed by the spherical waves and the rectangular reflectors.

When the simple source is moved out from the apex of the corner along the line of symmetry with respect to the three walls forming the corner, the power output reaches a minimum of about 7% of the free-field value at $r=0.49\lambda$. Thus the destructive interference is about 93% complete.

As this source is moved out from the corner, its output starts at 9 db above the free-field value, and then falls to 11.5 db below it in a distance of about $\lambda/2$. Thus at this point the drop in sound power output is 20 db from the initial value, a considerable reduction.

APPLICATION TO ENCLOSURES

The above results for the power output of a source as a function of position near reflectors were derived under the assumption that the reflecting surfaces were of infinite extent, and the question arises as to whether the results are valid when the surfaces form part of an enclosure, such as a reverberation chamber or an ordinary room. When a source radiates in an enclosure, the sound can be reflected back and forth repeatedly, and evidently the impedance reflected back onto the source may differ substantially from the amounts computed above for nonenclosing reflectors.

In the next section of this paper, experimental evidence is given which shows that the results are valid in a large reverberation chamber. Here we will mention physical considerations which indicate that the results

‡ See reference 2. Lamb's *Hydrodynamics* contains an error on this point (see reference 9, p. 498, lines 5 and 6).

[11] H. F. Olson, *Elements of Acoustical Engineering* (D. Van Nostrand Company, Inc., Princeton, 1947), p. 28.

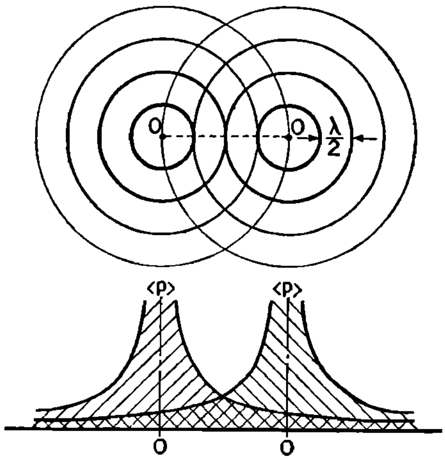

FIG. 7. Interference between the wave systems of two point sources. In the lower part of the figure the rms pressure $\langle p \rangle$ for each source is plotted along the line joining the two sources. Along this line $\langle p \rangle$ falls off according to an inverse first power law. λ is the wavelength.

are approximately true for any enclosure under two restrictions.

These restrictions are (a) that the surfaces of the enclosure other than those being considered as the reflectors must be distant at least a wavelength from the source, and (b) that the absorption in the enclosure must not be too small. Of course, the original limitations imposed on the reflecting surfaces associated with the source still hold, i.e., that these surfaces (c) must be rigid, and perfect or nearly perfect reflectors, and (d) must be large compared to the wavelength.

Under conditions (a) and (b), the reflected energy from the far walls of the enclosure (i.e., those walls not already considered to be associated with the source) will contribute little reflected impedance to the source.

As an example of the effect of a partial enclosure, we can consider the case of a simple point source equidistant from two plane parallel walls. The walls have a pressure reflection coefficient $R \leq 1$ which is independent of the angle of incidence of plane waves.

An analysis of this case is given in Appendix I. The power output of the source depends on the absorption of the walls, and the distance in wavelength separating the walls, as shown in Fig. 8. When the distance between the source and each wall exceeds the wavelength, the power output of the source differs from the free-field value by less than 1 db even when the wall absorption is low.

The image theory of the action of a reflecting environment on a source can be applied to a rectangular reverberation chamber, where the impedance reflected onto the source by the environment will vary with the distance of the source from the reflecting surfaces and the dissipation present. From physical considerations one would expect this to be the case whether the dissipation occurs at the walls or in the medium, and to be independent of room shape.

The next question to consider is whether quantitative criteria can be given for conditions (a) and (b) above. If the absorption of the enclosure is given, how many wavelengths away must reflecting surfaces be for the impedance they reflect on the source to be within given bounds?

Unfortunately, simple answers are possible only in simple cases, e.g., for a simple source near a corner, etc. It should be noticed that the directional pattern of the source is important here. The impedance reflected from a reflector onto a source with a pencil-shaped directional pattern is generally different from that reflected onto a source emitting spherical waves. For in the former case nearly all the sound radiated can be reflected back onto the source by a plane reflector several wavelengths away, while in the latter case only a small fraction can be so reflected.

The curves in Figs. 3, 5, and 8, and the experimental evidence in the next section indicate that for constant velocity sources of spherical waves, reflectors at distances greater than λ from the source will have small (less than 1 db or 25%) effect on the power output, even though the enclosure is very reverberant (e.g., average surface absorption coefficient about 3%).

For directional sources, the corresponding distance will depend on the orientation of the source, and may exceed λ. In such cases, and where the configuration of the reflecting surfaces is not simple, it may be easier to measure the variation of power output with position than to compute it.

EXPERIMENTAL RESULTS

Figure 9 gives some experimental evidence for these effects. The solid curve is a plot of Eq. (24), and gives the theoretical variation of the power output of a simple point source as it is moved away from two reflectors at right angles.

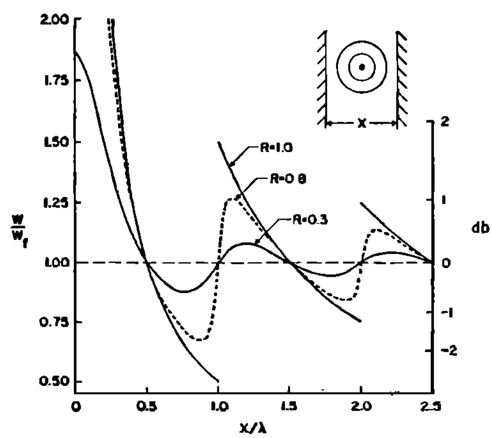

FIG. 8. Theoretical curve for the relative power output W/W_f of a simple point source equidistant from two plane parallel walls. x/λ is the distance in wavelengths between the walls. The walls have a pressure reflection coefficient $R \leq 1$ which is independent of angle of incidence for plane waves. The curve is a plot of Eq. (31).

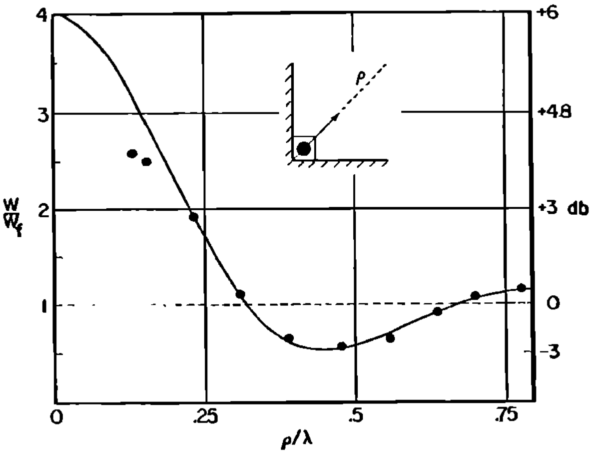

FIG. 9. Relative power output W/W_f of a simple source as a function of distance ρ along the line of symmetry from the origin at the vertex of a two-plane reflector. The solid line is the theoretical curve for a single frequency given by Eq. (24). The curve for a warbled frequency would be little different, see Fig. 5. The points are experimental data for a warbled signal 350±50 cps. The dashed line is the asymptote.

The black dots are experimental points, measured in the National Bureau of Standards reverberation chamber, two walls being used as the two reflectors.

Loudspeakers 4 in. in diam, mounted in small boxes filled with mineral wool, were used to simulate simple sources. They were driven with a warbled signal of frequency 350±50 cps, which gave a wavelength of about 3 ft, i.e., about 6 times the box diameter.

The voltage across each loudspeaker voice coil was kept constant throughout the experiment, in an attempt to make the loudspeaker cones vibrate with constant amplitude, as the theory required. The reasonable agreement obtained between theory and experiment (see the following, and Fig. 9) indicated that the loudspeakers used this way did resemble simple sources of constant strength, except for the near-field region.

One loudspeaker was put in each of the 4 lower corners of the reverberation chamber. The sound level in the chamber was measured by fixed microphones as the sources were moved along the floor away from the corners of the chamber. The measured sound levels gave the sound power output of the sources as functions of position; a uniform reverberant sound field in the central part of the chamber was assumed. Four sources were used instead of one to make the sound field more uniform. The walls of the reverberation chamber were brick, plastered and painted, giving a sound absorption coefficient of about 2% at this frequency.

Experimental points could not be obtained very near the corner owing to the physical dimensions of the loudspeaker box. The two experimental points shown nearest the corner are appreciably off the theoretical curve; this was probably caused by the departure of the loudspeaker field from the spherically symmetric field required by the theory. However, the rest of the data agree quite well with the theoretical curve.

In another experiment, one 4 in. diam loudspeaker was taken out of its box and used to simulate a dipole source. It turned out to be unnecessary to use 4 loudspeakers. A warbled frequency of 300±50 cps was used.

Figure 10 shows the results when the axis of the "dipole" was normal to a reflecting wall. The experimental curve is of the same type as that given by the theory for a dipole, and although there are significant differences between the 2 curves these can reasonably be attributed to the loudspeaker source differing from the ideal dipole source.

Similar experiments were made using 8 in. and 12 in. diam loudspeakers, and in all cases the main features of the theoretical curve were confirmed, but the details varied. It is quite entertaining to perform this experiment in a qualitative way, by moving a loudspeaker away from the wall of a reverberant room, the loudspeaker axis being normal to the wall. The increase in level from the minimum, obtained with the loudspeaker touching the wall, to the first maximum at a distance of about 0.3λ is quite striking.

Experiments with the axis of the loudspeaker dipole parallel to the wall were also performed, and give results which agreed with the theory in a similar way.

Additional confirmation that the theoretical results apply in a reverberation chamber is given by the experimental results already published[10] for the interference patterns that exist at the boundaries of a reverberant sound field.

In that case the source was fixed, and the microphone signal was measured as the microphone was moved along a certain path near the reflecting wall. If the chamber is kept constant, it follows from the theory of reciprocity that if the source and microphone are interchanged, and the source is now moved along the same path near the reflector, the microphone must record the same variation as it did before. This means the output of the source must vary with its position.

Thus the foregoing results for the source follow by reciprocity theory from the existence of the interference patterns in the reverberation chamber, subject to the restrictions that usually apply in reciprocity theory, for example that the source, microphone, and any absorbers present are linear.

An interesting example of reciprocity is that the variation in signal picked up by a velocity microphone when moved normal to a reflecting wall is the same as the variation in output of a dipole source moved along the same path, excluding the very-near-field region. This can be seen by comparing Fig. 4 and Eq. (21) of this paper with Fig. 9 and Eq. (6) on p. 253 of reference 10. Both the velocity microphone and the dipole source have the same figure-of-eight directional pattern, the first as a receiver and the second as an emitter. The orientation of this directional pattern with respect to the wall was the same in both cases, of course.

MEASUREMENT OF THE POWER OUTPUT OF A SOUND SOURCE

In considering some practical consequences of these effects, we first inquire how far one assumption used in deriving the above results will hold in practice. The assumption was that the volume velocity or vibration of the source was independent of changes in the impedance it worked into.

Apparently this is true for most sound sources met in practice, such as transformers, jet engines, appliances, and loudspeakers, although it is hard to find published evidence on this point. Such sources are not matched into the medium, and are little affected by impedance changes within the limits we are considering here.

The internal impedance of such sources is much higher than the radiation impedance of the air they work into. Thus they act analogously to constant current sources in electricity and deliver power proportional to the load resistance.

A related fact is that such sources are inefficient sound generators. A transformer whose whole mass is vibrating, dissipates much more energy in internal friction than in sound radiation, and its motion is evidently largely independent of the latter.

At the present time there is some interest in the measurement of the sound power output of sources; an American Standards Association standard on this topic is in preparation. In this paper we have shown that such measurements may be affected by the presence of a reflector in four ways. A reflector can change the power output, the directional pattern, the extent of the near field, and the radiation impedance of a sound source.

It is clear then that the position of the source relative to reflectors must be carefully considered in measuring its sound power output. One can measure the free-field value, or some other value near a reflector, or perhaps both. As an example, if a transformer is always used mounted on a concrete slab it is probably most useful to measure its power output in that position, and not bother with its free-field output.

In measuring the power output of a simple source by the reverberation chamber method, the source can in principle be placed anywhere in the reverberation chamber, and the results corrected to the free-field value (or the value corresponding to any other position near plane reflectors at right angles) by using the equations in Table II. However, the correction varies with frequency, and the composite correction for the power output of a broad-band source might be laborious to compute. Also the results would be restricted to simple sources.

Thus in practice if the free-field output of a source is required, it is probably most convenient to place the source and microphone(s) far enough away from all reflecting surfaces (walls, floors, ceilings, vanes, etc.) in the chamber for these interference effects to be negligible. For simple sources, the errors from these effects will in general be less than 1 db, if the source and microphones are placed at least $\lambda/2$ from the nearest boundaries, and at least 2λ, say, from the other boundaries of the chamber. See reference 10, p. 254.

For a source with a nonspherical-directional pattern of sound radiation, the corresponding distances will depend on the orientation of the source, and may exceed these. In such cases it may be more convenient to find the variation of power output with position by experiment than by calculation.

ACKNOWLEDGMENT

The author wishes to thank the members of the National Bureau of Standards, particularly Dr. R. K. Cook, for helpful discussions of various points in this paper.

APPENDIX I. OUTPUT OF A SIMPLE SOURCE BETWEEN TWO PARALLEL WALLS

A simple source is equidistant from two-plane parallel walls whose separation is x, and whose pressure reflection coefficient $R \leq 1$ is independent of the angle of incidence of plane sound waves.

At a point a small distance r from the source, on a line making an angle θ with the normal from the source to the reflectors, $r \ll x$ and $r \ll \lambda$, the potential caused by the direct and reflected waves is

$$\psi = (1/r)\cos(\omega t - kr) + (R/x)[\cos(\omega t - kx + kr\cos\theta) + \cos(\omega t - kx - kr\cos\theta)] + (R^2/2x)[\cos(\omega t - 2kx + kr\cos\theta) + \cos(\omega t - 2kx - kr\cos\theta)] + \cdots \quad (28)$$

$$= (1/r)\cos(\omega t - kr) + 2(A\cos\omega t + B\sin\omega t)\cos(kr\cos\theta) \quad (29)$$

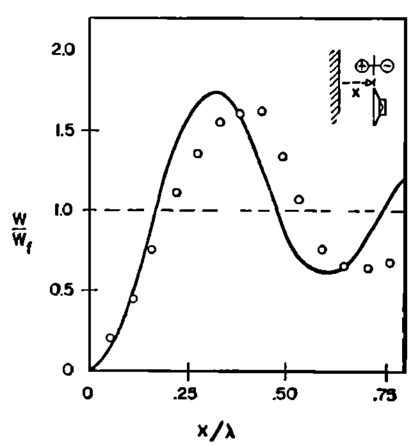

FIG. 10. The curve is taken from Fig. 4. The circles are experimental points measured in a reverberation chamber, for the relative power output W/W_f of a dipole source (axis normal to reflector) *vs* distance x/λ in wavelengths from a plane reflector. See text.

where

$$A=\sum_{n=1}^{\infty}(R^n/nx)\cos nkx$$

and

$$B=\sum_{n=1}^{\infty}(R^n/nx)\sin nkx. \quad (30)$$

From (29) we obtain the pressure $p=-\rho_0(\partial\psi/\partial t)$ and the radial particle velocity $v=\partial\psi/\partial r$. Next the mean value of pv, i.e. $\langle pv\rangle$, is taken, and this expression is integrated over the surface of the sphere of radius r. Then the power output of the source is

$$W=\lim_{r\to 0}4\pi r^2\int_0^{\pi/2}\langle pv\rangle\sin\theta d\theta$$

$$=2\pi\rho_0\omega k(1+2B/k)$$

i.e.,

$$W/W_f=1+(2/kx)\tan^{-1}[(R\sin kx)/(1-R\cos kx)],\quad R<1, \quad (31)$$

where W_f is the output of the source in a free field.

For $R=1$,

$$W/W_f=1+(\pi/kx),\quad 0<kx<2\pi$$
$$=1+(3\pi/kx),\quad 2\pi<kx<4\pi,\quad \text{etc.}$$

The solution is finite for all nonzero values of kx; the energy escapes between the walls. For $R=1$, the value of W/W_f jumps discontinuously from $\frac{1}{2}$ to $\frac{3}{2}$ at $kx=2\pi$, and jumps from $\frac{3}{4}$ to 5/4 at $kx=4\pi$, etc.

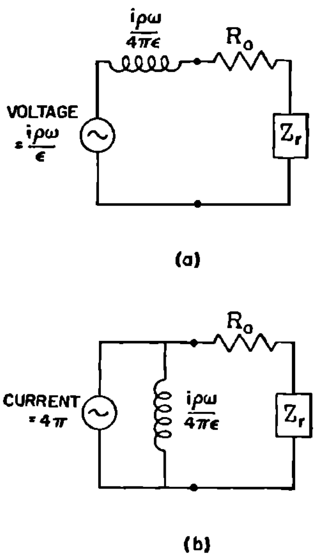

FIG. 11. Equivalent circuits for a sound source consisting of a small pulsating sphere of radius $\epsilon\ll\lambda$. Z_r is the reflected impedance caused by a plane reflector. In the $\lim_{\epsilon\to 0}$ the source becomes a simple point source.

Equation (31) is plotted in Fig. 8. Stenzel[12] gives expressions in the form of infinite series for the velocity potential in some more general cases of this type.

APPENDIX II. IMPEDANCE REFLECTED ONTO SIMPLE SOURCE BY PLANE REFLECTOR

The output of two point sources was calculated by Rayleigh[5] by finding the mean-squared pressure at a point in the far field, and integrating this over a spherical surface. A different method, based on the fact that the reaction of a source (or a reflector) on another source can be considered as a reflected impedance, is as follows.

We consider first a source similar to a simple point source, but with a small finite radius ϵ; the source is thus a pulsating sphere, with volume velocity independent of ϵ. The center of this sphere is distant x from a plane rigid reflector, $\epsilon\ll x$, $\epsilon\ll\lambda$.

The potential ψ at a distance r from the source center, $\epsilon<r\ll x$, λ, can then be written

$$\psi=(1/r)\exp i(\omega t-kr)+(1/2x)\exp i(\omega t-2kx+kr\cos\theta). \quad (32)$$

The corresponding pressure and velocity are given by $p=-\rho(\partial\psi/\partial t)$, $v=(\partial\psi/\partial r)$, and the impedance seen by the source is

$$Z=p/4\pi r^2v \quad \text{at} \quad r=\epsilon \quad (33)$$

$$=(\rho\omega k/4\pi)[1+(\sin 2kx/2kx)+i(1/k\epsilon+\cos 2kx/2kx)], \quad (34)$$

dropping second-order terms. Thus $Z=Z_0+Z_r$, where Z_0 is the free-field value of the impedance, and Z_r is the reflected impedance. We then have

$$Z_0=\lim_{x\to\infty}Z \quad (35)$$

$$=(\rho\omega k/4\pi)(1+i/k\epsilon) \quad (36)$$

$$=R_0+iX_0. \quad (37)$$

Equation (34) is equivalent to the electrical circuit shown in Fig. 11(a) which can be transformed to that shown in Fig. 11(b). In the latter, the mass reactance proportional to $1/k\epsilon$ is shunted across the source and becomes infinite as the source approaches a point source of the same volume velocity, i.e., as $\epsilon\to 0$; the source then works into the free-field radiation resistance R_0, due to the medium, in series with the reflected impedance Z_r.

From (36), $R_0=\rho\omega k/4\pi$, and the free-field power output of the source is $\langle I^2\rangle R_0$, where $\langle I^2\rangle$ is the mean-

[12] H. Stenzel, Ann. Physik 43, 1–31 (1943).

squared volume velocity. The reflected impedance, due to the presence of the reflector, is

$$Z_r = R_r + iX_r \tag{38}$$

$$= (\rho ck^2/4\pi)[j_0(x') + in_0(x')] \tag{39}$$

$$= (\rho ck^2/4\pi)h_0(x'), \tag{40}$$

where $j_0(x')$ and $n_0(x')$ are spherical Bessel function of the first and second kind, and $x' = 2kx$; $h_0(x')$ is the spherical Hankel function. A plot of R_r *vs* X_r gives the usual impedance spiral, see Fig. 12. If the reflecting wall is not rigid, but gives a "pressure release" boundary condition ($p=0$), the corresponding reflected impedance is the same as (40) but negative.

It is emphasized that these impedances[13] apply only to a simple point source. The same reflector will in general reflect a different impedance onto a different type of source, for example, a line source.

The power output of the point source in the presence of the reflector can be calculated in three different ways. (1) By integrating $\langle p^2 \rangle$, the mean-squared pressure, over a surface in the far field which encloses the source. (2) By integrating $\langle pv_{\mathrm{normal}} \rangle$ over any convenient surface enclosing the source, as was done in Appendix I.

[13] G. Laville and T. Vogel, Acustica 7, 101 (1957).

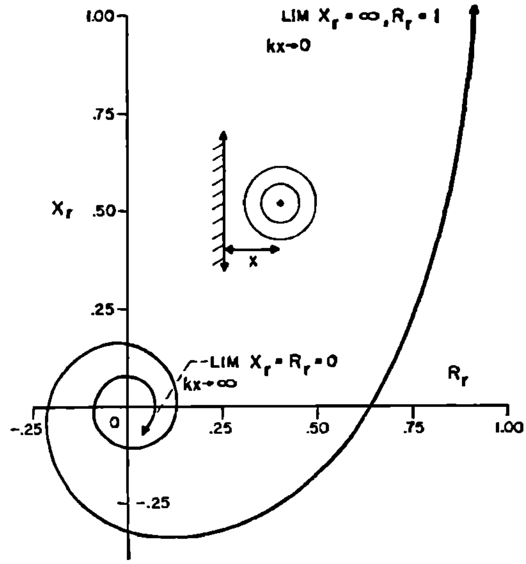

FIG. 12. Impedance diagram showing how the impedance $Z_r = R_r + iX_r$, reflected from a plane reflector onto a simple source, varies with kx.

(3) By taking the product $\langle I^2 \rangle \mathrm{Re}[\lim_{r\to 0}(p/4\pi r^2 v)]$ as above. From (34) this gives

$$\langle I^2 \rangle (\rho\omega k/4\pi)[1 + (\sin 2kx/2kx)] = W_f[1 + j_0(x')]$$

the same as (6).